Supplementary Material :
Fluttering Pattern Generation using
Modified Legendre Sequence for Coded Exposure Imaging

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As the supplementary material, we provide the derivation of Eq.7 in the original paper as well as closer looks at the experimental results.

1. Derivation of Equation 7

Let $\hat{U}$ be a fluttering shutter pattern with the elements $\hat{u}_i \in \{0, 1\}$. We denote B as $B = \hat{U} - \mu$ with elements $b_i \in \{-\mu, 1-\mu\}$, where μ is the mean value of the elements in $\hat{U}$. Then we introduce $U = 2(B + \mu - 0.5)$, where $u_i \in \{-1, 1\}$. The difference between $\hat{U}$ and U is that the sequence values have changed from $\{0, 1\}$ to $\{-1, 1\}$.

Let $\hat{a}_k$ be the autocovariance of $\hat{U}$ and t_k be the autocorrelation of B, then $\hat{a}_k = t_k$ [1]. We denote a_k as the autocorrelation of U, which is derived as follows.

[1] P. Boufounos. Generating binary processes with all-pole spectra. In *Proceeding of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2007*

$$
\begin{aligned}
a_k &= \sum_{i=0}^{n-k-1} u_i u_{i+k} \\
&= 4 \sum_{i=0}^{n-k-1} (b_i + m)(b_{i+u} + m) \quad (\text{where} \quad m = \mu - 0.5) \\
&= 4 \sum_{i=0}^{n-k-1} (b_i b_{i+k} + m b_i + m b_{i+k} + m^2) \\
&= 4 \sum_{i=0}^{n-k-1} b_i b_{i+k} + 4 \sum_{i=0}^{n-k-1} (m^2 + m b_i + m b_{i+k})
\end{aligned}
$$

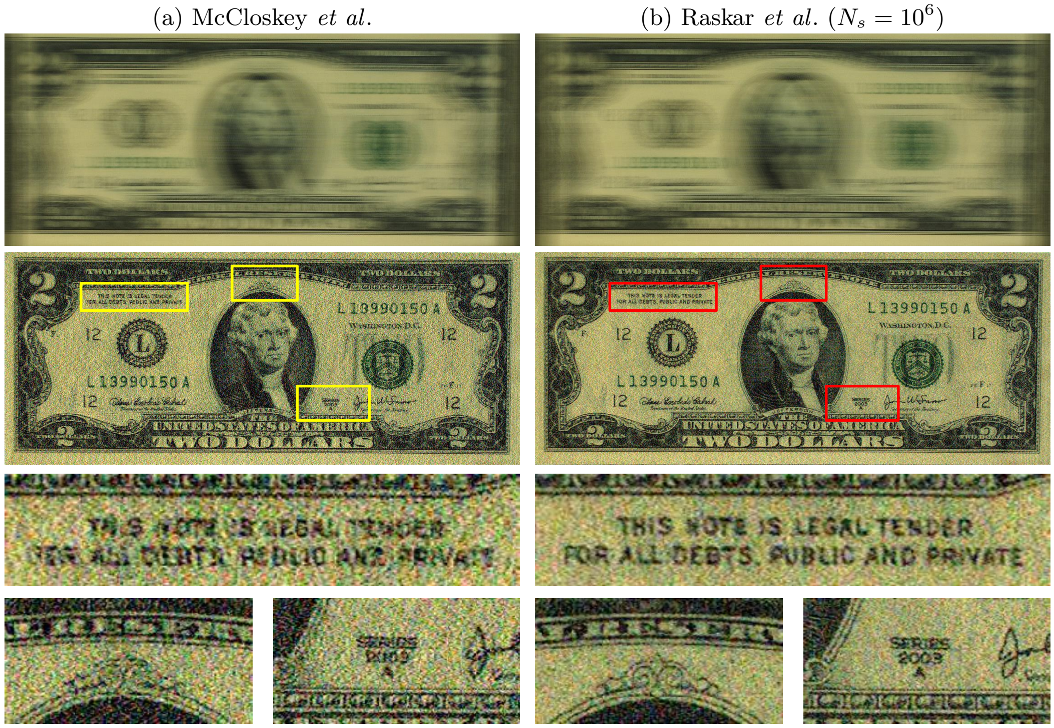
$$
\begin{aligned}
&= 4 \sum_{i=0}^{n-k-1} b_i b_{i+k} + 4(\sum_{i=0}^{n-k-1} m^2 + m \sum_{i=0}^{n-k-1} (b_i + b_{i+k})) \\
&= 4 t_k + 4((n-k)m^2 + m \sum_{i=0}^{n-k-1} (b_i + b_{i+k})) \\
&= 4 \hat{a}_k + 4m \sum_{i=0}^{n-k-1} (\hat{u}_i + \hat{u}_{i+k} + 3\mu - 0.5) \quad (\text{where} \sum_{i=0}^{n-k-1} b_i b_{i+k} = \hat{a}_k)
\end{aligned}
$$

As mentioned in the original paper, m becomes 0 with the assumption that the sequence is balanced with equal number of zeros and ones for optimal autocorrelation properties.

2. High-Resolution Results

(a) McCloskey *et al.*

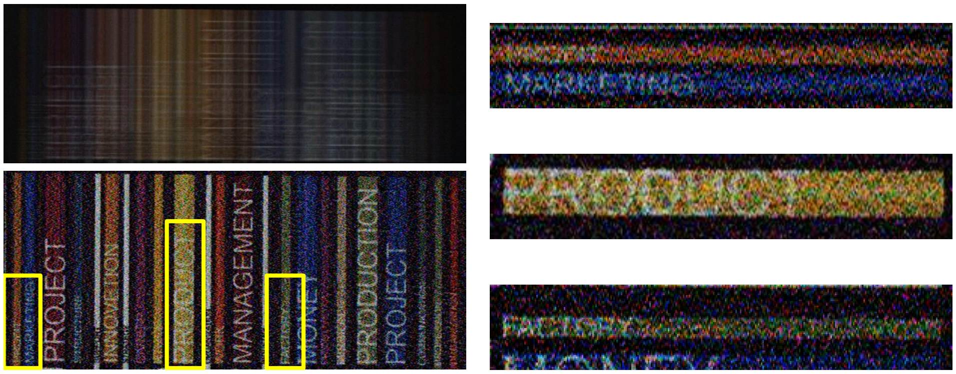
(b) Raskar *et al.* ($N_s = 10^6$)



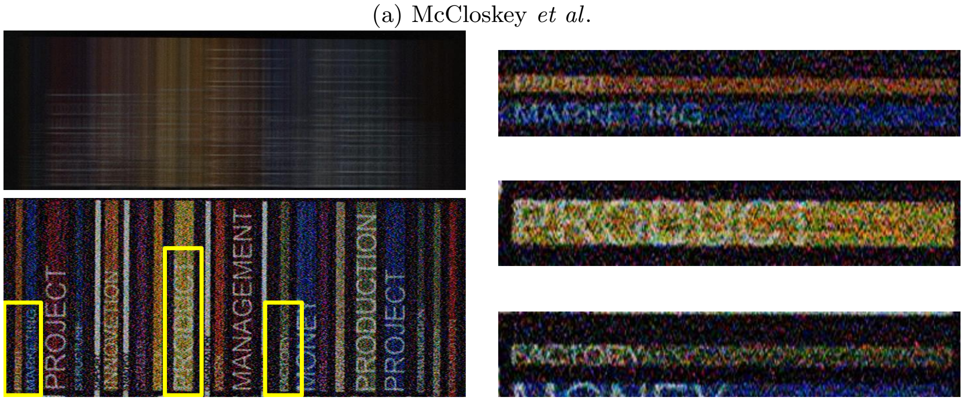
(c) Raskar *et al.* ($N_s = 10^8$)

(d) Proposed

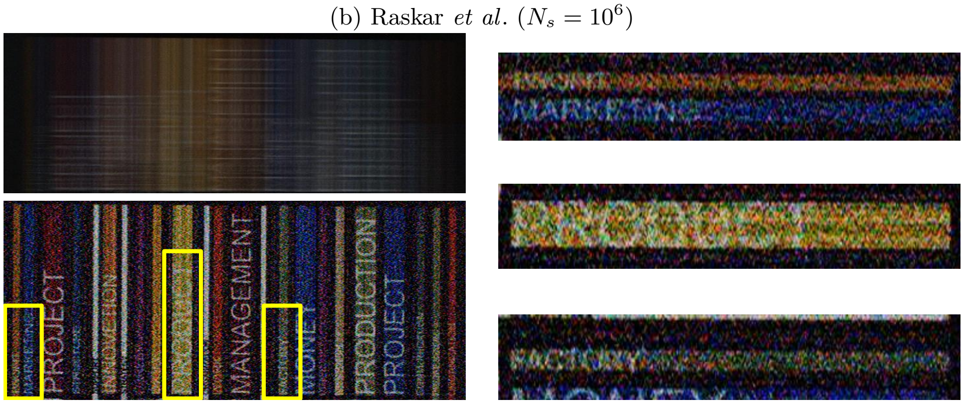
Figure 1: Comparison of the deblurring performance with the fluttering patterns of length 130 generated by different methods. The proposed sequence suppresses deconvolution noise and preserves edges of the deblurred image better than the other sequences.



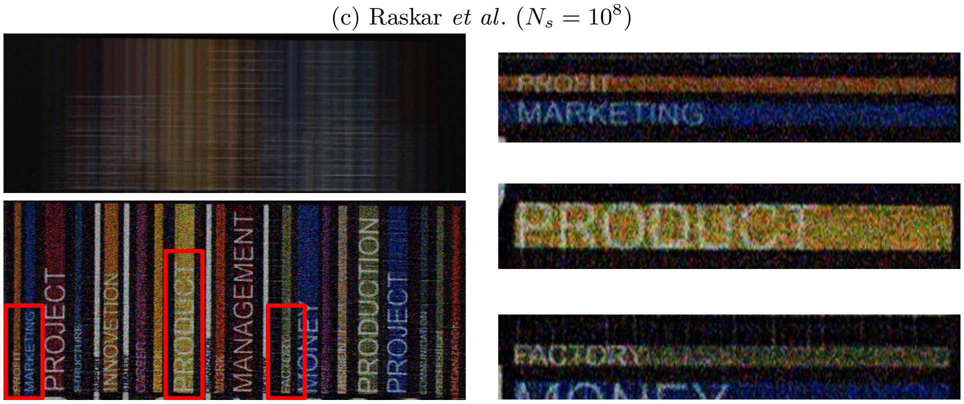
(a) McCloskey *et al.*



(b) Raskar *et al.* ($N_s = 10^6$)



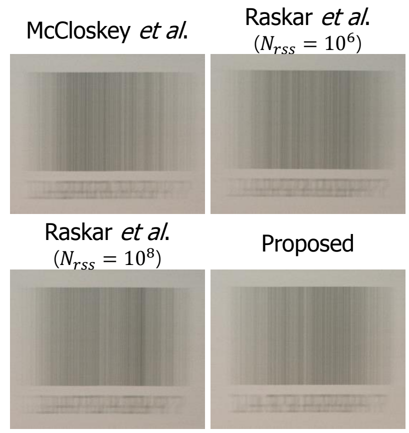
(c) Raskar *et al.* ($N_s = 10^8$)



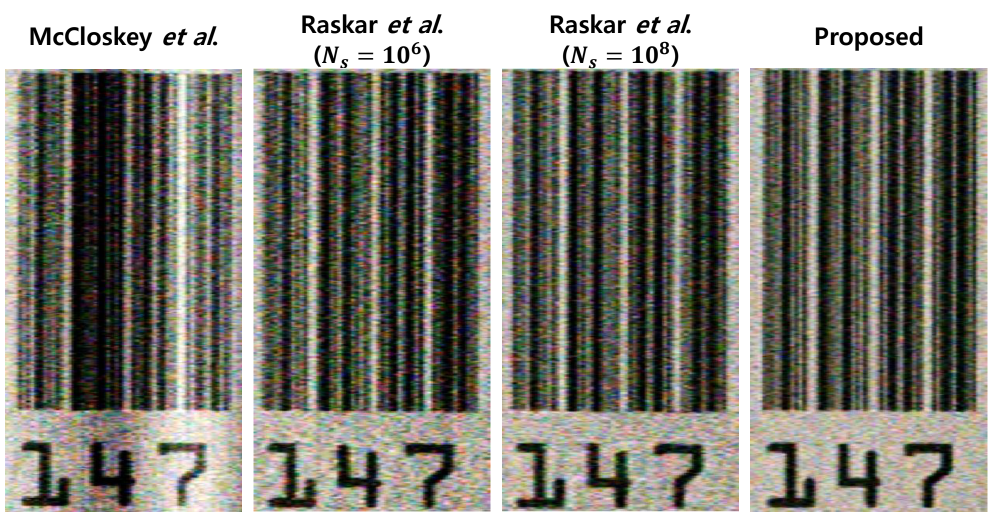
(d) Proposed

Figure 2: Comparison of the deblurring performance with the fluttering patterns of length 100 generated by various methods.

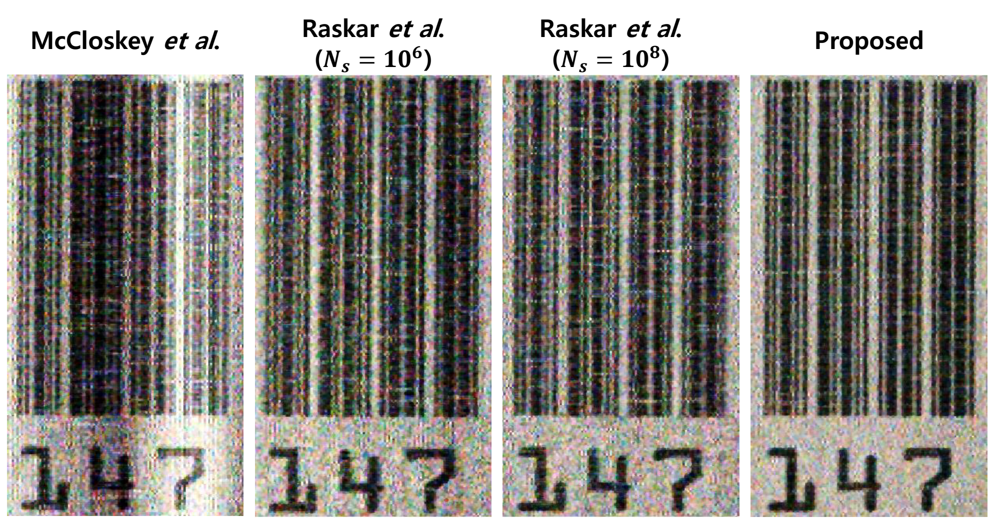
(a) Static image



(b) Blurred images



(c) Deblurred images

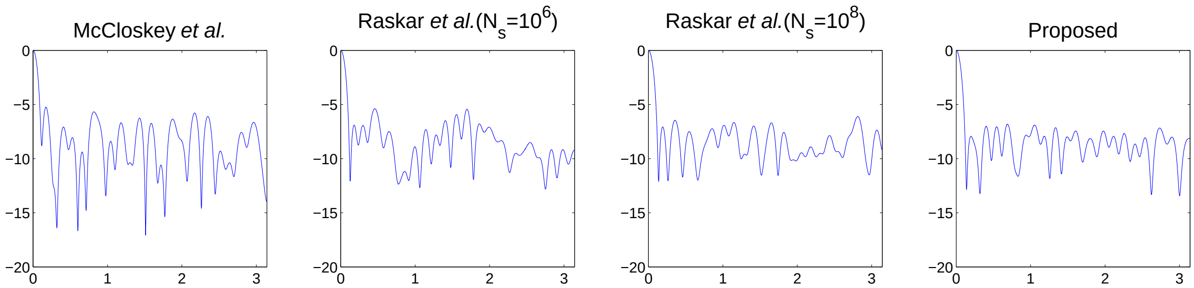


(d) Resolution enhanced images from motion blur

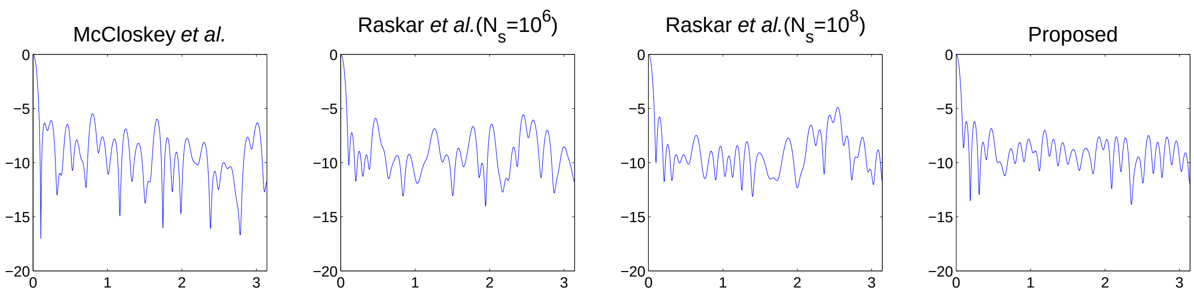
Figure 3: Comparison of the resolution enhancement performance. (a) Static image of a barcode. (b) Captured images with different fluttering patterns of length 120. (c) Bicubic upsampled images by two after deblurring. (d) Resolution enhanced images using motion blur. In (c, d), the results with the proposed sequence are clearer than results with the other sequences.

3. MTF plot

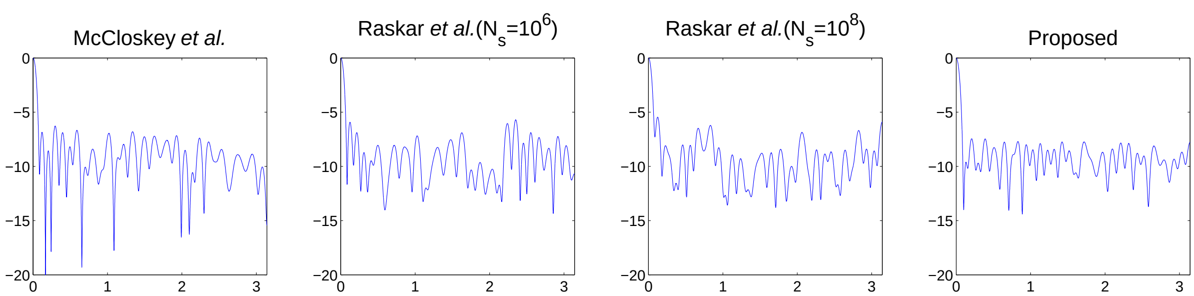
The following plots compare the MTFs of fluttering sequences of different length computed using different methods. It can be clearly seen that the results using our method show the desired properties the best, that is, a MTF of a good binary sequence should be flat with maximal minimum value.



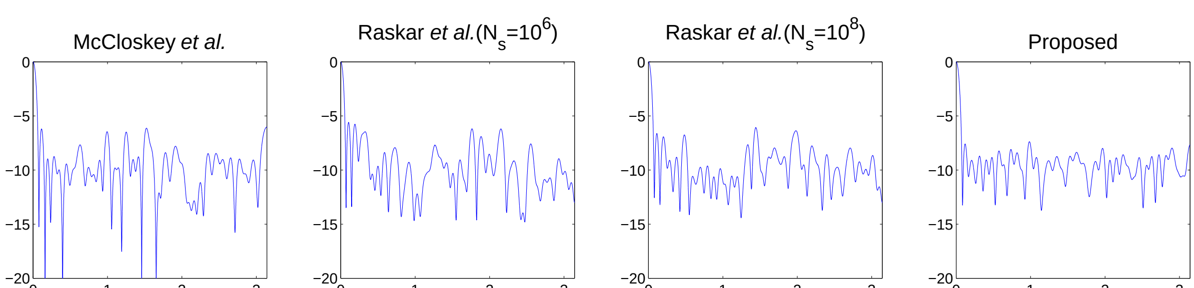
(a) Sequence Length 50



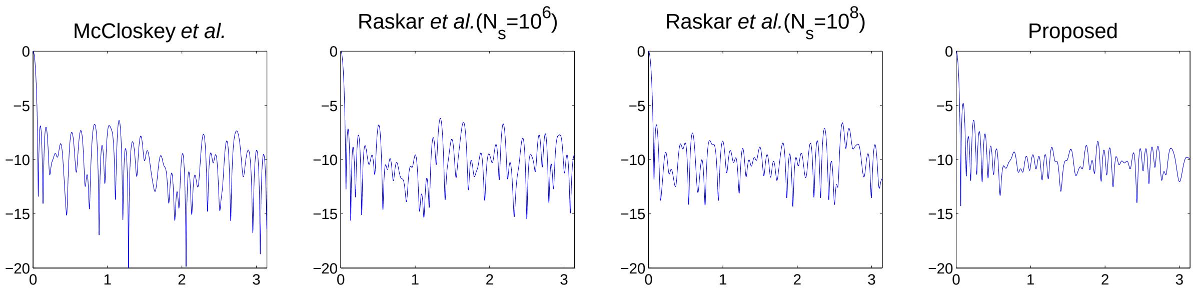
(b) Sequence Length 60



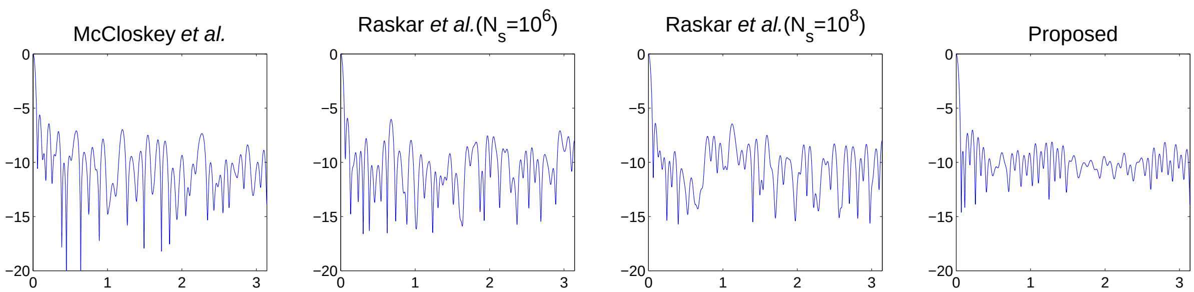
(c) Sequence Length 70



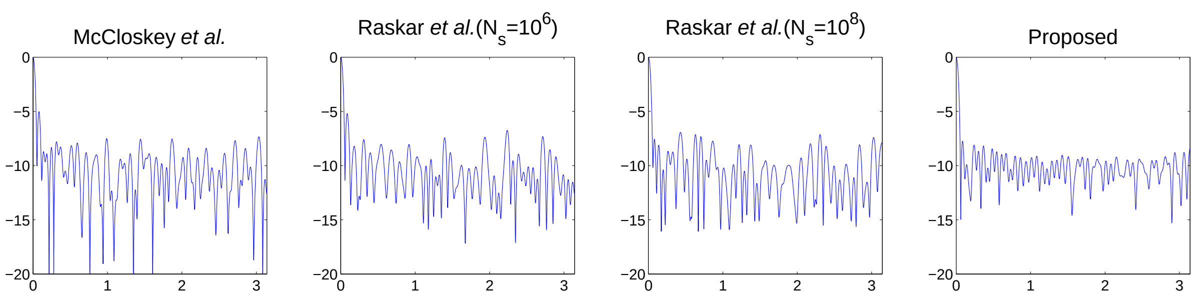
(d) Sequence Length 80



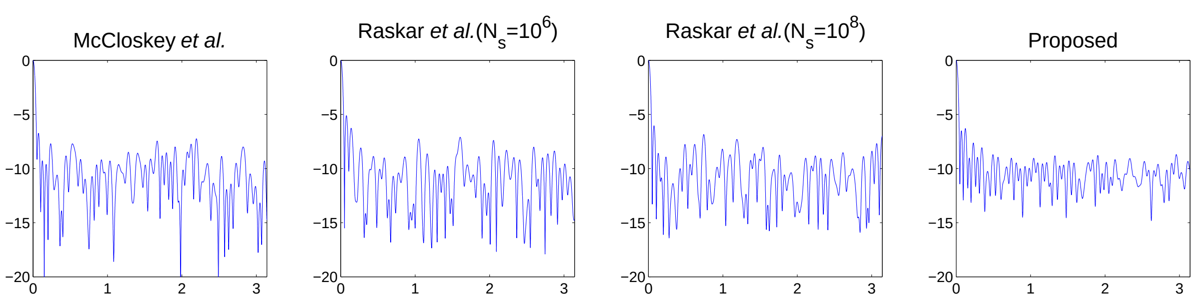
(a) Sequence Length 90



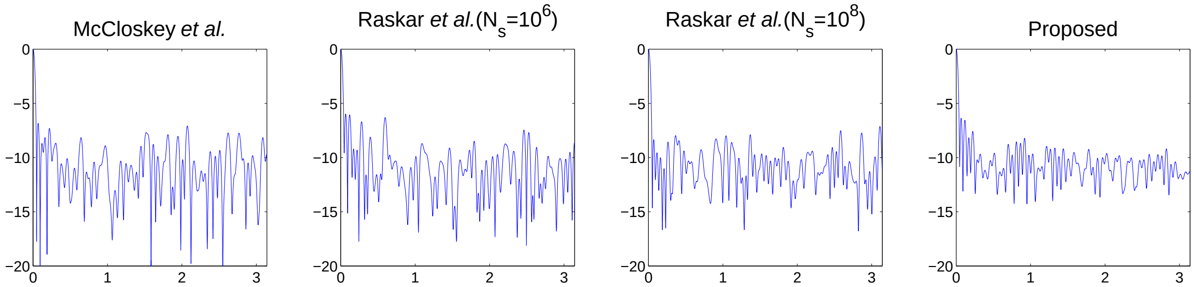
(b) Sequence Length 100



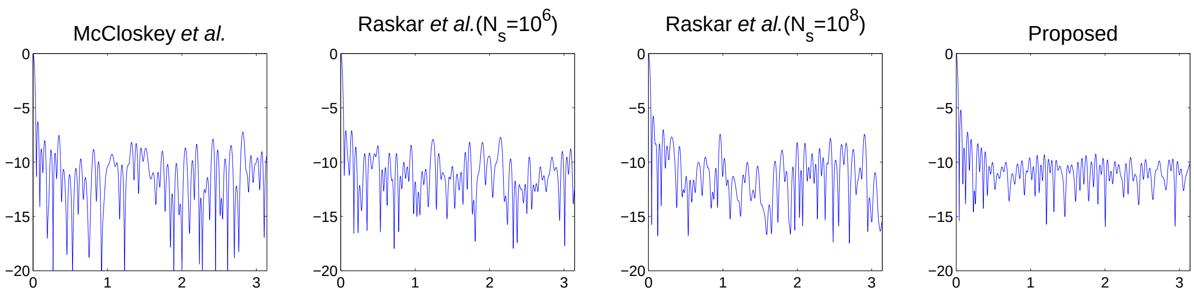
(c) Sequence Length 110



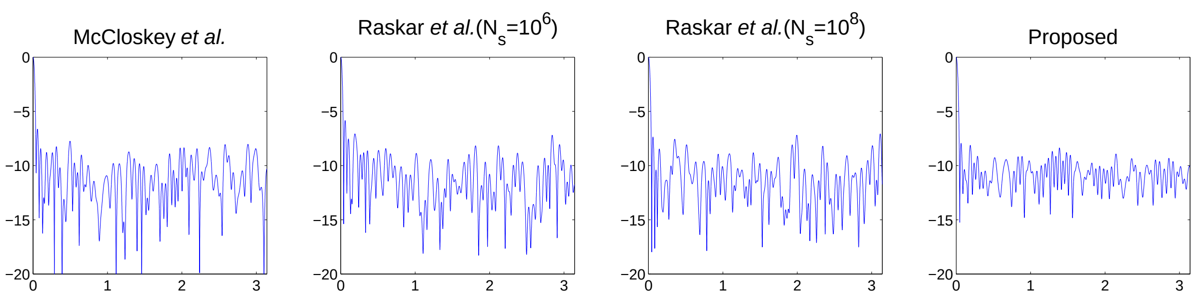
(d) Sequence Length 120



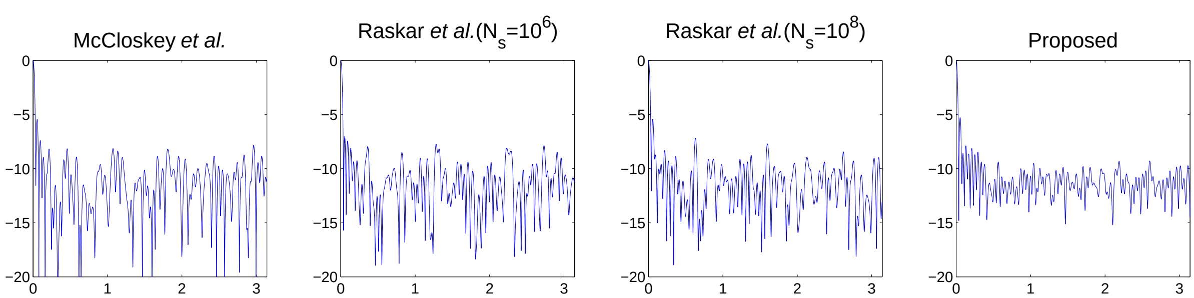
(a) Sequence Length 130



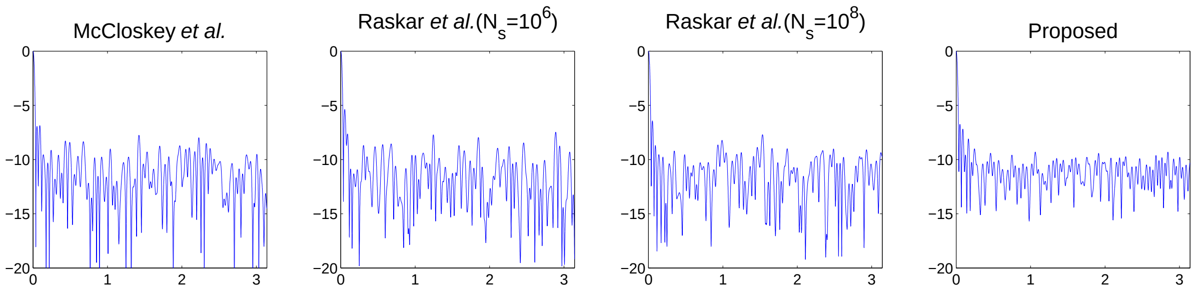
(b) Sequence Length 140



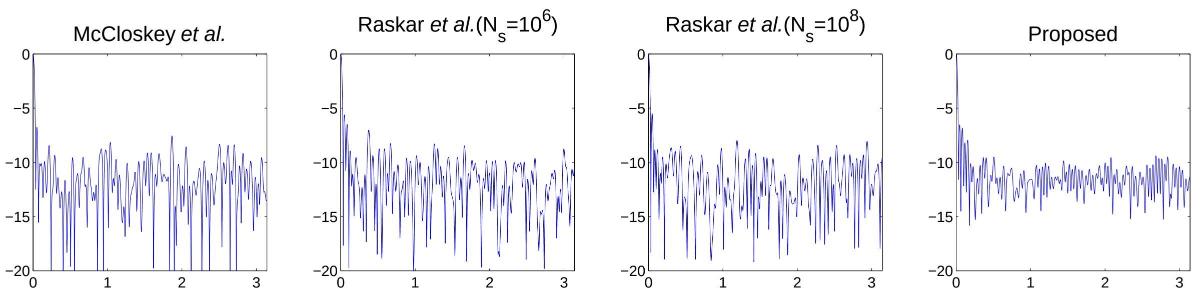
(c) Sequence Length 150



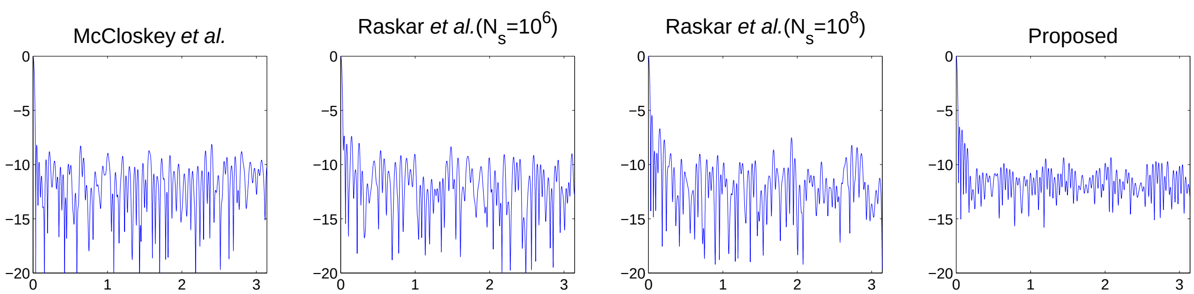
(d) Sequence Length 160



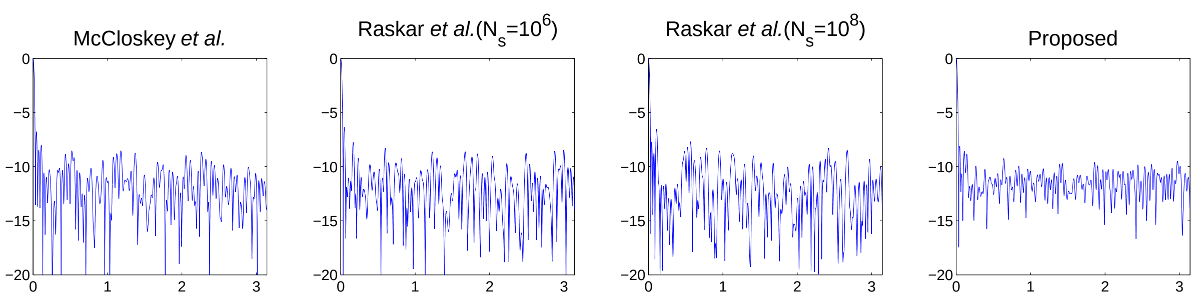
(a) Sequence Length 170



(b) Sequence Length 180



(c) Sequence Length 190



(d) Sequence Length 200