

Fluttering Pattern Generation using Modified Legendre Sequence for Coded Exposure Imaging

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Abstract

Finding a good binary sequence is critical in determining the performance of the coded exposure imaging, but previous methods mostly rely on a random search for finding the binary codes, which could easily fail to find good long sequences due to the exponentially growing search space. In this paper, we present a new computationally efficient algorithm for generating the binary sequence, which is especially well suited for longer sequences. We show that the concept of the low autocorrelation binary sequence that has been well exploited in the information theory community can be applied for generating the fluttering patterns of the shutter, propose a new measure of a good binary sequence, and present a new algorithm by modifying the Legendre sequence for the coded exposure imaging. Experiments using both synthetic and real data show that our new algorithm consistently generates better binary sequences for the coded exposure problem, yielding better deblurring and resolution enhancement results compared to the previous methods for generating the binary codes.

1. Introduction

Image deblurring is one of the most traditional problems in computer vision, in which the goal is to recover a latent sharp image from an image blurred due to the motion of the subject or the camera. Although the solution for the image deblurring problem has been sought for the last few decades, it still remains as a challenging problem.

An approach that has shown promising deblurring results is to tackle the problem in an active manner by changing the way images are captured in a camera. The technique, which is called the coded exposure photography [19], flutters the camera's shutter open and close in a special manner within the exposure time in order to preserve the spatial fre-



(a) Raskar *et al.* [19] ($N_s = 10^8$)

(b) Proposed

Figure 1: Comparison of the deblurring performance with the fluttering patterns of length 130 generated by different methods.

quencies, thereby simplifying the deblurring problem to be invertible.

A key element in the coded exposure imaging is the generation of the fluttering pattern of the shutter (binary sequence). In previous work, a near optimal binary sequence is computed through a randomized linear search [1, 2, 19] or a priority search [16] over the space of potential sequences. While these methods are applicable for generating short sequences, they are not suitable for computing long binary sequences because of the large search space.

Finding binary sequences with low autocorrelation is a deeply studied problem in the field of information theory and physics because it relates to many applications in telecommunications (e.g., synchronization, pulse compression and, especially, radar), physics (e.g., Ising spin glasses) and chemistry [6]. Among many methods for generating the binary sequence, the Legendre sequence has shown some advantages over other methods, especially in terms of the

computational time and the autocorrelation measure [9].

In this paper, we introduce a new algorithm for generating binary sequences for the coded exposure imaging using the Legendre sequence. We show that the concept of the low autocorrelation binary sequence can be applied for generating the fluttering pattern of the shutter, and propose a new algorithm that modifies the Legendre sequence to make it suitable for the coded exposure problem. Our new algorithm consistently generates better binary sequences for the coded exposure problem in much shorter time (several orders of magnitude), yielding better deblurring and resolution enhancement results compared to the previous methods for generating the binary codes (Fig. 1). Our algorithm holds significant advantages especially when the sequence length is large, in which case the previous methods fail to find good binary codes due to the exponentially growing search space.

The remainder of the paper is organized as follows. After reviewing previous work in Section 2, we introduce the coded exposure imaging concept and explain the measure of a good binary sequence developed in the field of Information Theory as well as its adaptation to coded exposure in Section 3. Then a modified Legendre sequence for coded exposure is proposed in Section 4. The performance of our algorithm is evaluated in Section 5 and we conclude with discussions in Section 6.

2. Previous Work

Image deblurring is a classic problem in computer vision that has been actively studied over the last several decades. Traditional solutions to the problem include Richardson-Lucy [15, 20] and Wiener filter [26], but several new directions have been explored recently to enhance the deblurring performance. Fergus *et al.* [5] took a statistical approach in a variational Bayesian framework by using a natural image prior on image gradients, while Shan *et al.* [23] incorporated spatial parameters to enforce natural image statistics using a local ringing suppression. In [13], Levin *et al.* proposed a solution for defocus blur by using the coded aperture and the sparse natural image prior to produce sharper edges and reduce undesirable ringing artifacts. We refer to [14] for a comprehensive review of the deblurring literature.

In [19], Raskar *et al.* introduced the coded exposure photography, a motion deblurring method using the fluttered shutter. Rather than having the shutter open for the entire exposure duration, they flutter the camera’s shutter open and closed during the exposure with a binary pseudo-random sequence [19]. With the fluttered shutter, spatial details in the blurred image are preserved, making the deconvolution a well-posed problem. Tai *et al.* presented a spatially varying PSF estimation algorithm which jointly utilizes a coded exposure camera and simple user interactions in [24], while McCloskey *et al.* further addressed the problem of motion

deblurring using the coded exposure by analyzing the design and the estimation of the coded exposure PSF in [17]. The idea of coded exposure photography has also been extended to the resolution enhancement application in [1].

As expected, the fluttering pattern of the camera shutter plays a critical role in determining the performance of the coded exposure imaging. Agrawal and Xu proposed a method for finding optimized codes for both PSF estimation and invertibility in [2]. McCloskey presented the idea that the shutter sequence must be dependent on the object velocity and proposed a method for computing the velocity-dependent sequences in [16]. To actually compute the binary sequence, previous works rely on either a random sample search [1, 2, 19] or a priority search [16] over the space of potential sequences. Natural image statistics are incorporated in generating binary sequences for coded aperture [27] and coded exposure [17]. While the search based methods are serviceable for short sequences, they are computationally infeasible for long sequences because of the large search space. To overcome this limitation, we introduce an algorithm for computing binary sequences suited for long sequences using Legendre sequence.

3. Measure of a Good Binary Sequence

Assuming spatially-invariant motion blur, the blur process is modeled as follows:

$$\mathbf{B} = \mathbf{A}\mathbf{I} + \mathbf{n}, \tag{1}$$

where \mathbf{B} , \mathbf{I} and \mathbf{n} represent the blurred image, the latent un-blurred image, and the noise, respectively. The matrix \mathbf{A} is called the smearing matrix, which describes the convolution of the latent input image with a point spread function.

The principal idea behind the coded exposure is to improve the invertibility of the imaging process (the invertibility of the smearing matrix \mathbf{A}) through the fluttered shutter (see Fig. 2 (a)(b)). Denoting a binary sequence of length n as $U = [u_0, \dots, u_{n-1}]$, a near-optimal binary code is computed through a randomized linear search with the following conditions in [19]:

- (i) $\operatorname{argmax}_U \min(|\mathcal{F}(U)|)$,
- (ii) $\operatorname{argmin}_U \operatorname{var}(|\mathcal{F}(U)|)$ or $\operatorname{argmin}_U \operatorname{mean} (A^T A)^{-1}$

where $\mathcal{F}(U)$ is the discrete Fourier transform of the binary sequence and its absolute value $|\mathcal{F}(U)|$ is a magnitude of frequency response of binary sequence (MTF: Modulation Transfer Function). The condition (i) relates to preserving the spatial frequency in a blurred image and the condition (ii) describes the variance of the MTF or the deconvolution noise.

3.1. Merit Factor

As mentioned earlier, finding binary sequences is also of importance in the field of information (coding) theory. In information theory, the *merit factor* is widely used as the criterion of “goodness” for binary sequences whose aperiodic autocorrelations are collectively small. For a binary sequence $U = [u_0, u_1, \dots, u_{n-1}]$, the merit factor $M(U)$ is defined as follows:

$$M(U) = \frac{n^2}{2 \sum_{k=1}^{n-1} a_k^2}, \quad (2)$$

where a_k is the *aperiodic autocorrelation* at shift k given by

$$a_k = \sum_{i=0}^{n-k-1} u_i u_{i+k}. \quad (3)$$

The merit factor is closely related to the signal to self-generated noise ratio, which corresponds to the deconvolution noise in the coded exposure imaging. In [10], the relation between the merit factor and the spectral properties of the sequence is denoted as

$$\sum_{k=1}^{n-1} a_k^2 = \frac{1}{2} \int_0^1 [|\mathcal{F}(U)|^2 - n]^2 d\theta. \quad (4)$$

For a fixed sequence of length n , Eq. (4) shows that the merit factor measures how much the amplitude spectrum of the sequence deviates from the constant value n , therefore a sequence with a higher merit factor has a flatter MTF. This corresponds to condition (ii) of the binary sequence measure for coded exposure and we can rewrite the merit factor as follows:

$$M(U) = \frac{n^2}{\int_0^1 [|\mathcal{F}(U)|^2 - n]^2 d\theta} \simeq \frac{n^2}{\text{var}(|\mathcal{F}(U)|)}. \quad (5)$$

3.2. Coded Factor

While the merit factor is a good criterion for measuring the MTF variance, it can make the amplitude spectrum partially peaky as shown in Fig. 2(c), which prevents the system from preserving the details of a blurred image. To deal with this problem, we define a new measure called the *coded factor* (F_C) to measure the quality of a binary sequence for coded exposure imaging:

$$F_C(U) = M(U) + \lambda \min[\log(|\mathcal{F}(U)|)], \quad (6)$$

where λ is the weighting parameter for balancing two terms and \log is used for normalizing the scales between the two terms.

We should note that Eq. (3) is derived with the binary sequence taking the value $\{-1, 1\}$. However, the binary sequence for the coded exposure should take the value $\{0, 1\}$,

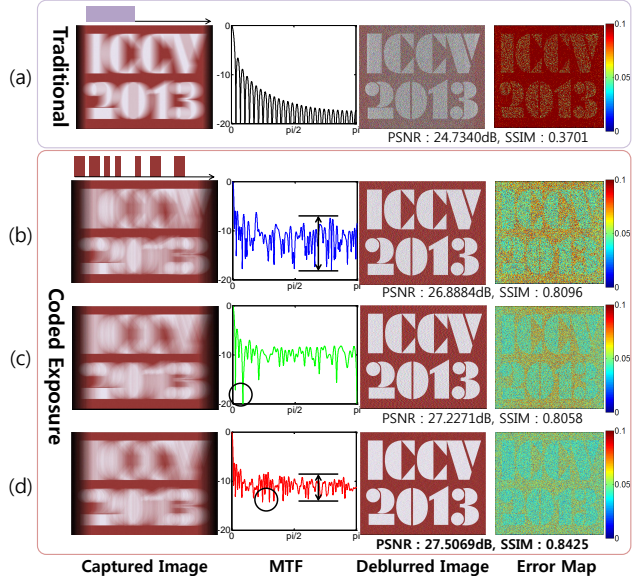


Figure 2: Coded exposure and the measure of a good binary sequence. (a) In a traditional camera, some information are lost at frequencies with MTF value of 0, making the deblurring problem an ill-posed problem. (b) By using the coded exposure, the information are preserved and the deblurring problem becomes invertible. (c) The merit factor is a good measure of the variance of the MTF (deconvolution noise), but it may make the spectrum partially peaky. (d) The proposing *coded factor* is a good measure of a binary sequence that minimizes the variance of the MTF while maximizing the lowest MTF value.

since the value -1 is physically infeasible. If we change the sequence $u_i \in \{-1, 1\}$ to $\hat{u}_i \in \{0, 1\}$ by substituting 0 for -1, the aperiodic autocorrelation of U is computed as

$$a_k = 4\hat{a}_k + 4\hat{m} \sum_{i=0}^{n-k-1} (\hat{u}_i + \hat{u}_{i+k} + 3\mu - 0.5), \quad (7)$$

where \hat{a} and μ represent the *autocovariance* and the mean of \hat{U} , respectively. The derivation of Eq. (7) is provided in the supplementary material. In Eq. (7), $\hat{m} = \mu - 0.5$, which becomes 0 with the assumption that the sequence is balanced with equal number of zeros and ones for optimal autocorrelation properties [12]. Therefore, we use the following equation for computing the merit factor from a binary sequence of 0’s and 1’s.

$$a_k \approx 4\hat{a}_k. \quad (8)$$

4. Modified Legendre Sequence for Coded Exposure

Although the importance of both terms in Eq. (6) are addressed in [19], a solution for finding a good binary se-

quence that simultaneously satisfies both conditions is not provided. Instead, they rely on a randomized linear search that only considers $\min[\log(|\mathcal{F}(U)|)]$ in Eq. (6). To deal with this issue, we find a solution that would take both terms into account and return the maximum coded factor F_C . For this, we turn to the Legendre sequence.

The Legendre sequence [7] is a binary sequence with a high merit factor and it is among the most popular choices for generating binary sequences in many different fields. The Legendre sequence of a prime length n is defined as

$$u_i = \begin{cases} 1 & \text{if } i = 0, \\ \left(\frac{i}{n}\right) & \text{if } i > 0, \end{cases} \quad (9)$$

where u and i represent an element value and index of the sequence, respectively. $\left(\frac{i}{n}\right)$ is the *Legendre symbol* that takes the value 1 if i is a *quadratic residue* modulo n and the value 0 otherwise¹.

Advantages of using the Legendre sequence over a random binary sequence search for the coded exposure include higher quality sequences with high merit factor as well as much less computational load since the Legendre sequence is solved in a closed form. Although the Legendre sequence would insure high merit factor $M(U)$, it does not guarantee the highest coded factor since it does not consider $|\mathcal{F}(U)|$. Therefore, to further improve the quality of the Legendre sequence for the coded exposure imaging, we propose an algorithm for generating a modified Legendre sequence by applying three sequence operations: *rotating*, *appending*, and *flipping* to find the sequence with the maximum *coded factor* $F_C(U)$ in Eq. (6).

Rotating. The merit factor of a Legendre sequence can be improved by rotating the sequence in a cyclic manner as shown in [8]. For a given sequence U , an r -rotated Legendre sequence V^r is defined as

$$V^r = (U_{r+1:n}; U_{1:r}), \quad (10)$$

where $U_{i:j}$ is the sub-sequence of U from the i^{th} to the j^{th} element and $(;)$ represents an operator for concatenating two sequences. We search for the enhanced sequence in terms of the *coded factor* among all candidate sequences V^r ($0 \leq r \leq n - 1$).

Appending. In [4], Borwein *et al.* proved that appending the initial part of a rotated Legendre sequence to itself could improve the merit factor of the sequence. We adopt the appending operation to improve the sequence quality as well as to resolve a restraint of Legendre sequence that it is only defined for a length of a prime number.

From an r -rotated Legendre sequence V^r , a t -appended Legendre sequence is obtained by appending the first t

¹Note that either $\{0, 1\}$ or $\{-1, 1\}$ can be used to represent the sequence value as shown in [18].

($0 \leq t \leq n - 1$) elements of the sequence to itself, and it is denoted as $Y^t = (V^r; (V^r)_{0:t-1})$. Using the sequence appending operation, the modified Legendre sequence with a length m can be generated from any rotated Legendre sequence with a prime length n for $\frac{m}{2} \leq n \leq m$.

Flipping. In a recent work [3], Baden presented an efficient optimization method of the merit factor of binary sequences by deriving a formulation for measuring the change in the merit factor by a change of value in an element (flipping) in the sequence. The formulation is given by

$$\delta_j = -8y_j((\Lambda \star Y)_j + (\Lambda \star Y^\gamma)_{m+1-j}) + 8(Y \star Y^\gamma)_{m+1-2j} + 8(m-2), \quad (11)$$

where δ_j is the change in the autocorrelation due to the flipping of the element j , \star represents the convolution operator, $\Lambda = [a_1, \dots, a_m]$ is an aperiodic autocorrelation of the binary sequence Y of length m , and γ indicates the reversal of a sequence where $y_j^\gamma = y_{m-j+1}$.

From Eq. (11), a candidate set of element indices that are expected to improve the merit factor is chosen as

$$\Delta_1 = \{j | \delta_j > 0\}. \quad (12)$$

To extend the above optimization method to the *coded factor*, we compute the change in the minimum of MTF by a single-element flip as follows:

$$\kappa_j = \begin{cases} \min |\mathcal{F}(Y) + \mathcal{F}(y_j)| - \min |\mathcal{F}(Y)| & \text{if } y_j = 0, \\ \min |\mathcal{F}(Y) - \mathcal{F}(y_j)| - \min |\mathcal{F}(Y)| & \text{if } y_j = 1, \end{cases}$$

$$\text{s.t. } \mathcal{F}(y_j) = e^{-i\omega(j+0.5)} \frac{2}{\omega} \sin \omega, \quad (13)$$

where $\mathcal{F}(y_j)$ is the DFT of a single element y_j . Thus, the candidate set Δ_2 is determined by

$$\Delta_2 = \{j | \kappa_j > 0\}. \quad (14)$$

The two sets Δ_1 and Δ_2 are then combined to construct a new candidate set Δ ($\Delta = \Delta_1 \cup \Delta_2$). Since Δ_1 is related to the merit factor and Δ_2 is related to the MTF minimum, the new candidate set Δ includes potential element indexes that can improve the coded factor F_C (Eq. 6). To determine the elements to flip among the candidates, we apply a variant of the *steep decent* algorithm in [3], which is described in Algorithm 1. Since the number of candidates in Δ is usually small, the computational load for Algorithm 1 is small.

Algorithm Summary Our framework for generating a binary sequence for the coded exposure imaging is summarized in Algorithm 2. To generate a sequence of length m , we first generate Legendre sequences with length n , which is a collection of prime numbers in the range between $\frac{m}{2}$ and

Algorithm 1 Optimization by *Flipping*

```
1: procedure OPTIMIZESEQUENCE(sequence =  $Y$ )
2:    $\hat{Y} = Y$ 
3:    $F_C(\hat{Y}) =$  coded factor of  $\hat{Y}$ 
4:    $\Delta =$  candidate set of  $\hat{Y}$ 
5:    $n_\Delta =$  the number of candidates in  $\Delta$ 
6:   for  $i = 1$  to  $n_\Delta$  do
7:      $F_C(Y') = \max(F_C)$  with  $i$ -bits flipping in  $\Delta$ 
8:     if  $F_C(Y') > F_C(\hat{Y})$  then
9:        $\hat{Y} = Y'$ 
10:    go to line 3
11:   end if
12: end for
13: return  $\hat{O}$ 
14: end procedure
```

Algorithm 2 Sequence Generation for Coded Exposure

```
1: procedure GENERATESEQUENCE(length =  $m$ )
2:    $\mathbf{n} = \{n_i | \frac{m}{2} \leq n_i \leq m, n_i \text{ is prime number}\}$ 
3:   for all  $n_i$  in  $\mathbf{n}$  do
4:      $U_i =$  Legendre Sequence of length  $n_i$  in Eq. (9)
5:      $V_i = \text{Rotating}(U_i)$ 
6:      $Y_i = \text{Appending}(V_i)$ 
7:   end for
8:    $O = Y_i$  with the highest coded factor
9:    $\hat{O} =$  Optimization by Flipping( $O$ ) in Algorithm 1
10:  return  $\hat{O}$ 
11: end procedure
```

m ($\mathbf{n} = \{n_i | \frac{m}{2} \leq n_i \leq m, n_i \text{ is prime number}\}$). We then find the r_i -rotated Legendre sequence using the *rotating* operation for each Legendre sequence and apply the *appending* operation for all the rotated sequences to make length m sequences. Among the candidate sequences, we select a sequence with the highest *coded factor* and perform the optimization using the *flipping* operation. An example of MTF changes according to each sequence operation is shown in Fig. 3.

5. Experiments

To evaluate the performance of the proposed algorithm, we conducted many coded exposure deblurring experiments using both synthetic and real-world dataset. We compare our results with the results obtained by using the method by Raskar et al. in [19] and by McCloskey et al. in [17]. For our method and the method [19], binary sequences of length [40, 50, \dots , 200] were generated. We used the code by the author² to generate the sequences for the method [19] as well as to deblur the images. The number of random

Method	TIME (seconds)
Raskar et al. [19] ($N_s = 10^6$)	268.06
Raskar et al. [19] ($N_s = 10^8$)	26779.90
Proposed	0.21

Table 1: Average computational times for generating binary sequences. The computational time of the method [19] depends on the number of samples, while the proposed method requires much shorter time for all sequence lengths.

samples N_s of the method [19] was set to 10^6 and 10^8 to check the tradeoff between the computational time and the sequence quality. The average computational times for generating the sequences are shown in Table 1. The sequences for the method [17] were provided by the author for length [50, 60, \dots , 200]. The λ in Eq. (6) is set to 8.5 for all of our experiments. We use two deconvolution algorithms: (1) matrix inversion approach for the sake of comparing the performance of previous methods and the proposed method, and (2) non-blind deconvolution with hyper-Laplacian prior [11] to maximize the quality of the deblurred images. High resolution results and an executable are available online³.

5.1. Synthetic results

We performed synthetic experiments for quantitative evaluations. The synthetic data consist of 29 high quality images downloaded from Kodak Lossless True Color Image Suite [21]. Blurred images are simulated by 1D filtering with the binary sequences generated by each method and then adding intensity dependent Gaussian noise with a standard deviation $\sigma = 0.01\sqrt{i}$ where i is the noise-free intensity of the blurred images in [0, 1] [22]. The peak signal-to-noise ratio (PSNR) and the gray-scale structural similarity (SSIM) [25] are used as the quality metrics, which are calculated by averaging the results of 29 synthetic images.

To first show the effectiveness of the coded factor as a measure of a good binary sequence for coded exposure, we compared deblurring results using the binary sequences generated by the merit factor and the coded factor, which is shown in Fig. 4. The sequences generated by the coded factor shows stable performance while the sequences generated by the merit factor sometimes work poorly especially in terms of the SSIM due to the peaky spectrum as previously shown in Fig. 2(c).

Fig. 5 shows the comparisons of the deblurring results on the synthetic dataset. When the sequence length is relatively short, the method [19] performs slightly better than the proposed method. However, our method consistently produces good binary sequences for the coded exposure imaging and the difference in the performance amplifies as the sequence

²www.umiacs.umd.edu/~aagrwal/MotionBlur/SearchBestSeq.zip

³https://sites.google.com/site/jyleecv/legendre_coded_exposure

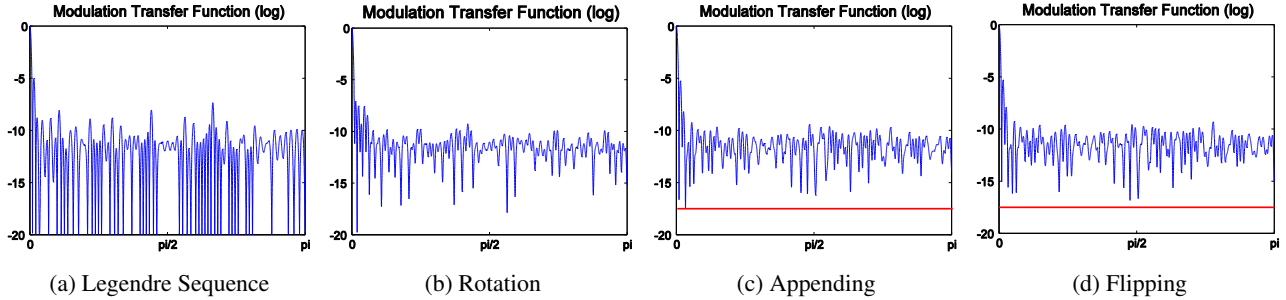


Figure 3: An example of MTF changes according to each sequence operation. The red lines in (c)&(d) indicate the minimum MTF of the sequence (c).

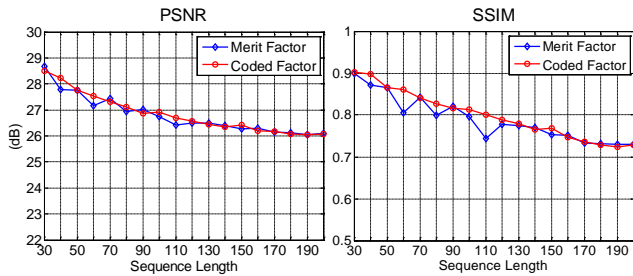


Figure 4: Comparison of the merit factor and the coded factor as a measure of a good binary sequence.

length increases. When the sequence length is big, the other methods fail to find good sequences due to the large search space. This issue of the sequence length is an important one since longer sequences are necessary for larger motion blurs or faster moving objects [1, 16]. The work in [1] specifically emphasized the need for finding good long sequences.

5.2. Real-world results

We implemented the coded exposure photography using the PointGrey Flea3 camera, which supports the *Trigger mode 5* that enables multiple pulse-width trigger with a single readout. In our implementation, each shutter chop is 1 ms long, so a fluttering pattern of length 100 has 100 ms capture time.

Fig. 1 and Fig. 6 show two examples of the deblurring results using the coded exposure with the fluttering patterns generated by various methods. As expected, deblurring results using the modified Legendre sequence returns the sharpest images, enabling the contents to be read as opposed to other results where the contents remain difficult to interpret. Fig. 7 shows another example of our coded exposure imaging in action, imaging static objects from a fast moving camera. By using the fluttering patterns that are generated by using our method, the scene contents become legible after deblurring, which otherwise would be very difficult to read.

The work in [1] showed that the coded exposure imag-



(a) Raskar [19] ($N_s = 10^6$) (b) Proposed

Figure 7: Our coded exposure imaging in action. Fluttering patterns of length 60 are used.

ing is not only effective for the motion deblurring but also for the resolution enhancement. In their analysis, the optimal code length is approximately $k * s$ for a given enhancement factor s and a blur size k . Therefore, they emphasized the importance of a long binary sequence as mentioned previously. Fig. 8 compares the performance of the resolution enhancement using different binary sequences of length 120, and as expected, the sequence generated by our method shows better visual quality in both the deblurring and the resolution enhancement.

In Fig. 9, we compare the deblurring performance with the same exposure time, but with different sequence lengths. The sequences of length 40 and 120 generated by the proposed method are used for this experiment and we control the single chop time to make the exposure time the same under different sequence lengths. As shown in Fig. 9, the deblurred image with the longer sequence preserves more spatial frequencies of the blurred image than the shorter sequence.

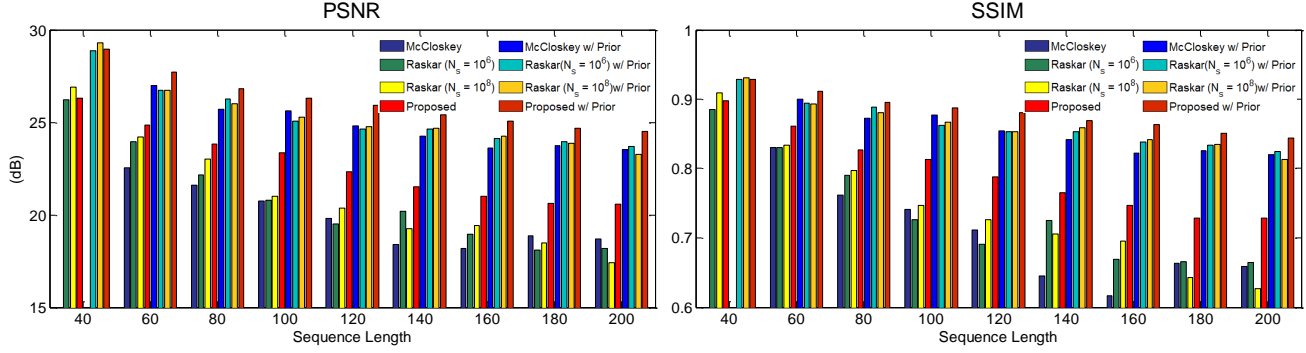
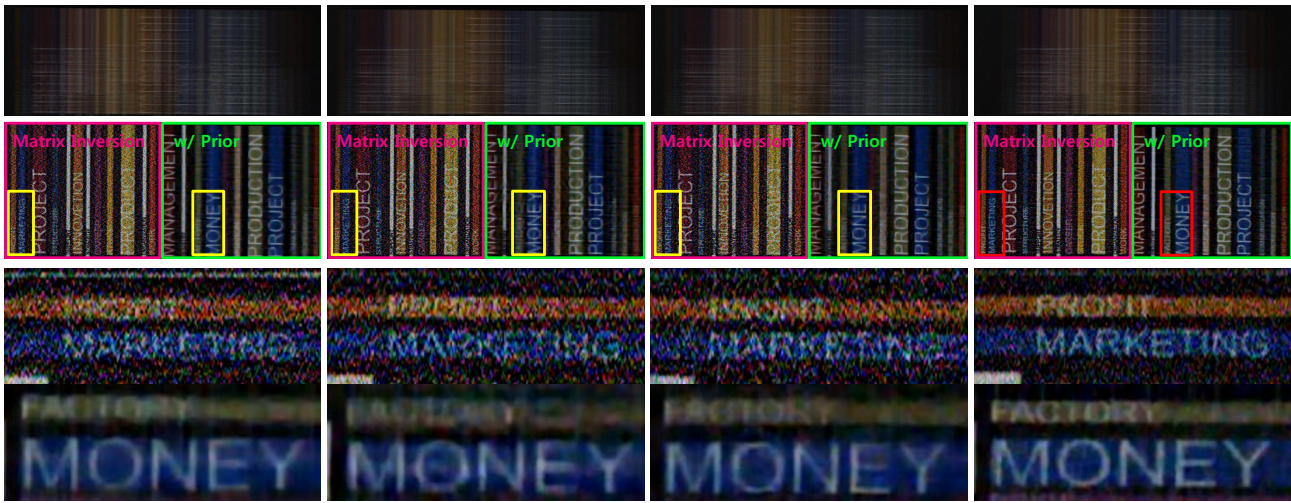
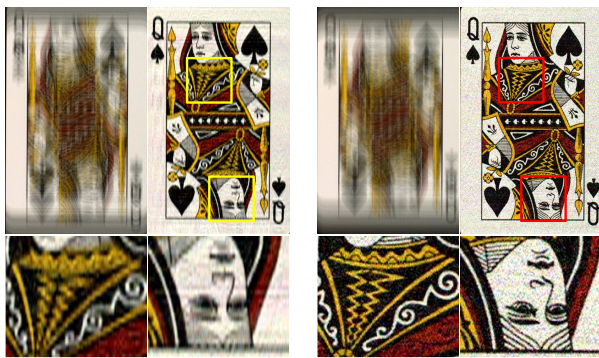


Figure 5: Comparison of the deblurring performance (synthetic): (left) PSNR and (right) SSIM. The difference in the performance amplifies as the sequence length increases; our method consistently generates good binary codes for the coded exposure imaging.



(a) McCloskey *et al.* (b) Raskar *et al.* ($N_s = 10^6$) (c) Raskar *et al.* ($N_s = 10^8$) (d) Proposed

Figure 6: Comparison of the deblurring performance with the fluttering patterns of length 100 generated by various methods. The blurred images are deblurred by using the matrix inversion method (Magenta) and the hyper-Laplacian prior [11] (Green).



(a) (b)

Figure 9: Comparison of the deblurring performance with different sequence lengths under the same exposure. (a) sequence length = 40, 1 chop duration = 3 ms. (b) sequence length = 120, 1 chop duration = 1 ms.

6. Discussion

We have presented a new method for computing the fluttering sequence for the coded exposure photography by modifying the Legendre sequence. We validated the efficiency of our algorithm through various experiments, and we were able to achieve better deblurring and resolution enhancement performance without any prior by using the binary codes generated using our algorithm. One of the biggest advantages of our algorithm is that we can compute binary sequences in near real time. In the future, we would like to take this advantage and extend our method to generate scene dependent fluttering sequences.

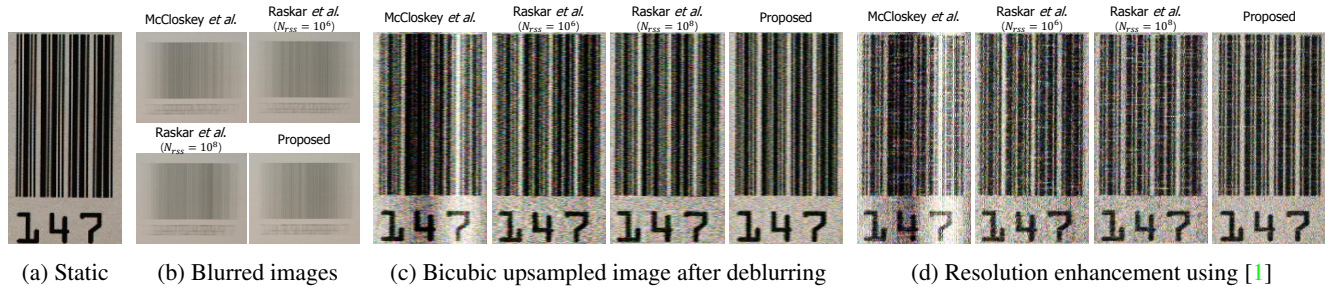


Figure 8: Comparison of the resolution enhancement performance. (a) Static image of a barcode. (b) Captured images with different fluttering patterns of length 120. (c) Bicubic upsampled images by two after deblurring. (d) Resolution enhanced images using motion blur. In (b, c), the results with the proposed sequence are clearer than results with the other sequences.

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